# Time Truncated Modified Chain Sampling Plan for Selected Distributions

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**ABSTRACT**: Multiple chain sampling plan is developed for a truncated life test when the life time of an item follows different life time distributions. The minimum sample sizes are determined when the consumer's risk and the test termination time are specified. The operating characteristic values for various quality levels are obtained and the results are discussed with the help of tables and examples.

**KEYWORDS:** Truncated life test, Marshall olkin extended exponential distribution, Generalized exponential distribution, Marshall – Olkin extended lomax distribution, Weibull distribution, Rayleigh distribution and Inverse Rayleigh distribution, Consumer's risk.

# I. INTRODUCTION

Quality and reliability engineering has gained its overwhelming application in industries as people become aware of its critical role in producing quality product for quite a long time, especially since the beginning of last century. It has been developed into a variety of areas of research and application and is continuously growing due to the steadily increasing demand. Acceptance sampling is major field of Statistical Quality Control (SQC) with longest history. Dodge and Romig, popularized it when U.S. military had strong need to test its bullets during World War II. If hundred percent inspection were executed in advance, no bullets would be left to ship. If, on the other hand, none were tested, malfunctions might occur in the field of battle, which may result in potential disastrous result.

Single sampling plans and double sampling plans are the most basic and widely applied testing plans when simple testing is needed. Multiple sampling plans and sequential sampling plans provide marginally better disposition decision at the expense of more complicated operating procedures. Other plans such as the continuous sampling plan, bulk-sampling plan, and Tighten-normal-tighten plan etc., are well developed and frequently used in their respective working condition.

Among these, Chain-sampling plans have received great attention because of their unique strength in dealing with destructive or costly inspection, for which the sample size, is kept as low as possible to minimize the total inspection cost without compromising the protection to suppliers and consumers. Some characteristics of these situations are

- (i) the testing is destructive, so it is favorable to take as few samples as possible,
- (ii) physical or resource constraint makes mass inspection on insurmountable task.

The original Chain sampling plan-1 (ChSP-1) was devised by Dodge (1977) to overcome the inefficiency and less discriminatory power of the Single sampling plan when the acceptance number is equal to zero. Two basic assumptions embedded with the design of chain sampling plans are independent process and perfect inspection, which means all the product inspected are not correlated and the inspection activity itself is error free. These assumptions make the model easy to manage and apply, though they are challenged as manufacturing technology advances.

In Dodge's approach, chaining of past lot results does not always occur. It occurs only when a nonconforming unit is observed in the current sample. This means that the available historical evidence of quality is not fully utilized. Govindaraju and Lai<sup>1</sup> developed a Modified Chain sampling plan(MChSP-1) that always utilizes the recently available lot-quality history. In a truncated life test, the units are randomly selected from a lot of products and are subjected to a set of test procedures, where the number of failures is recorded until the pre- specified time. If the number of observed failures at the end of the fixed time is not greater than the specified acceptance number, then the lot will be accepted. The test may get terminated before the pre-specified time is reached when the number of failures exceeds the acceptance number in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule we are interested in obtaining the smallest sample size to arrive at a decision, where the life time of an item follows different distributions. Two risks are continually associated to a time truncated acceptance sampling plan. The probability of accepting a bad lot is known as the producer's risk and the probability of rejecting a good lot is called the consumer's risk. An ordinary time truncated acceptance sampling plan have been discussed by many authors, Goode and Kao<sup>9</sup>, Gupta and Groll<sup>7</sup>, Baklizi and EI Masri<sup>2</sup>, Rosaiah and Kantam<sup>16</sup>, Tzong and Shou<sup>21</sup>, Balakrishnan, Victor Leiva

& Lopez<sup>4</sup>. All these authors developed the sampling plans for life tests using Single sampling plan. Sudamani ramaswamy A.R. and Sutharani.R<sup>11</sup>, (2013) ,discussed the Chain sampling plan for truncated life test using minimum angle method.

In this paper a new approach of designing Modified Chain sampling plan for truncated life test is proposed, assuming that the experiment is truncated at preassigned time ,when the lifetime of the items follows different distributions. The distributions considered in this paper are Marshall olkin extended exponential distribution, Generalized exponential distribution, Marshall – Olkin extended lomax distribution, Weibull distribution, Rayleigh distribution and Inverse Rayleigh distribution. The test termination time and the mean ratio's are specified. The design parameter is obtained such that it satisfies the consumer's risk. The probability of acceptance for MChSP-1 plan are also determined when the life time of the items follows the above distributions. The tables of the design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples.

### **II. GLOSSARY OF SYMBOLS**

Ν	-	Lot size
n	-	Size of the sample
d	-	Number of defectives in the sample
i	-	Acceptance criteria
P <sub>a</sub> (p)	-	Probability of acceptance of a lot submitted for inspection
$\mathbf{p}_0$	-	Failure probability
α	-	Producer's risk
β	-	Consumer's risk
σ	-	Scale parameter
λ,γ	-	Shape parameter
t	-	Prefixed time

#### **III. DISTRIBUTIONS**

The following are the distributions used in this paper:

#### (i) Generalized exponential distribution:

The cumulative distribution function (cdf) of the Generalized exponential distribution is given by

$$F(t,\sigma) = \left(1 - e^{-\frac{t}{\sigma}}\right)^{\lambda} , \quad t > 0, \sigma > 0$$
(1)

where  $\sigma$  is a scale parameter and  $\lambda$  is the shape parameter and it is fixed as 2.

#### (ii) Marshall – Olkin extended lomax distribution:

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The cumulative distribution function (cdf) of the Marshall - Olkin extended lomax distribution is given by

$$F(t,\sigma) = \frac{(1+\frac{t}{\sigma})^{\theta}-1}{(1+\frac{t}{\sigma})^{\theta}-\overline{\gamma}}, \quad \overline{\gamma} = 1-\gamma$$

$$, t > 0, \sigma > 0$$
(2)

where  $\sigma$  is a scale parameter and  $\theta$  and  $\gamma$  are the shape parameters and they are fixed as 2.

#### (iii) Marshall – Olkin extended exponential distribution:

The cumulative distribution function (cdf) of the Marshall – Olkin extended exponential distribution is given by

$$F(t,\sigma) = \frac{1 - e^{-t/\sigma}}{1 - \gamma e^{-t/\sigma}}, \quad \gamma = 1 - \gamma$$

$$(3)$$

where  $\sigma$  is a scale parameter and  $\gamma$  is the shape parameter and it is fixed as 2.

#### (iv) Weibull distribution:

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\sigma) = 1 - e^{-\left(\frac{t}{\sigma}\right)^{m}}, \quad t > 0, \sigma > 0$$
(4)

where  $\sigma$  is a scale parameter and  $\lambda$  is the shape parameter and it is fixed as 2. (v) **Rayleigh distribution:** 

The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t,\sigma) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}, t > 0, \sigma > 0$$
<sup>(5)</sup>

where  $\sigma$  is a scale parameter.

(vi) Inverse Rayleigh distribution:

The cumulative distribution function (cdf) of the Inverse Rayleigh distribution is given by

$$F(t,\sigma) = e^{-\frac{\sigma^2}{t^2}} , t > 0, \sigma > 0$$
(6)

where  $\sigma$  is a scale parameter.

#### IV. OPERATING PROCEDURE OF MODIFIED CHAIN SAMPLING PLAN

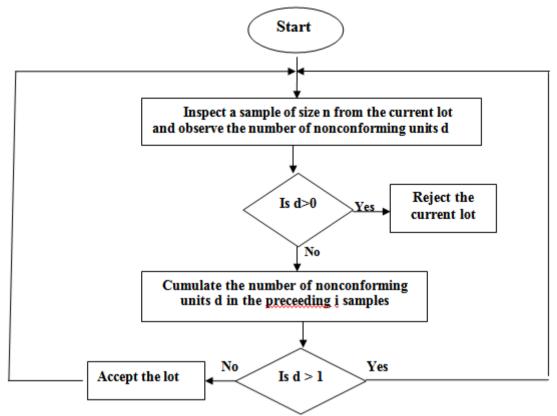
According to Govindaraju and Lai (1998), the operating procedure of MChSP-1 is as follows;

- (i) From each of the submitted lots, draw a random sample of size n. Reject the lot if one or more nonconforming units are found in the sample.
- (ii) Accept the lot if no nonconforming units are found in the sample, provided that the preceding i samples also contained no nonconforming units except in one sample, which may contain at most one nonconforming unit. Otherwise, reject thelot.

#### 4.1 Operating Procedure Of Modified Chain sampling Plan For The Life Tests

- (i) From each of the submitted lots, draw a random sample of size n. Reject the lot if one or more nonconforming units are found in the sample during the time  $t_0$ .
- (ii) Accept the lot if no nonconforming units are found in the sample during the time t<sub>o</sub>, provided that the preceding i samples also contained no nonconforming units except in one sample, which may contain at most one nonconforming unit. Otherwise, reject the lot.
- (iii)

# V. FLOWCHAR



Operating procedure of MChSP-1 plan for the life tests in the form of a flow chart.

# VI. CONSTRUCTION OF TABLES

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function of the sampling plan. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. We assume that the lot size is large enough to use the binomial distribution to find the probability of acceptance. The probability of acceptance for the sampling plan is calculated as follows

$$P_{a}(p) = P_{o,n}(P_{o,n}^{i} + iP_{o,n}^{i-1}P_{1,n})$$
------(7)

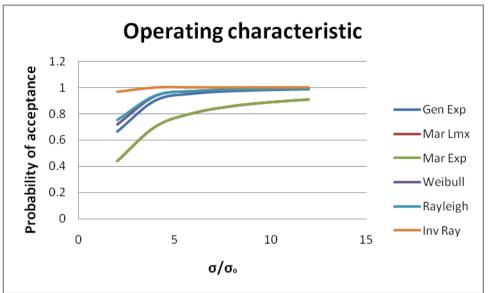
The time termination ratio  $t/\sigma_0$  are fixed as 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 4.712, the consumer's risk

 $\beta$  as 0.25, 0.10, 0.05, 0.01 and the mean ratios  $\sigma/\sigma_0$  are fixed as 2, 4, 6, 8, 10 and 12. These choices are consistent with Gupta and Groll <sup>7</sup>, Baklizi and EI Masri<sup>2</sup>, Balakrishnan et Al <sup>4</sup>. For various time termination ratios the design parameter values n are obtained by substituting the failure probability at the worst case in the equation (7) using the inequality

# $L(p_0) \leq \beta$

where  $p_0$  is the failure probability at  $\sigma = \sigma_0$  and are presented in Table 1 to Table 6. The probability of acceptance for MChSP- 1 sampling plan are also calculated for various time termination ratios and mean ratios and are presented in Table 7 to Table 12 for different life time distributions.

Figure1:



OC curve for Probability of acceptance, of MChSP-1 plan when the life time of an item follows different distributions

β	i	$t/\sigma_0$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	1	9	5	3	2	1	1	1	1
	2	4	2	1	1	1	1	1	1
	3	3	2	1	1	1	1	1	1
	4	2	1	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
0.10	1	14	8	5	3	2	1	1	1
	2	7	4	2	2	1	1	1	1
	3	5	2	1	1	1	1	1	1
	4	3	2	1	1	1	1	1	1
	5	3	2	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
0.05	1	18	10	6	4	3	2	1	1
	2	9	4	3	2	1	1	1	1
	3	6	3	2	1	1	1	1	1
	4	4	2	1	1	1	1	1	1
	5	3	2	1	1	1	1	1	1
	6	3	1	1	1	1	1	1	1
0.01	1	26	14	9	7	4	3	2	2
	2	13	7	4	3	2	1	1	1
	3	8	4	3	2	1	1	1	1
	4	6	3	2	1	1	1	1	1
	5	5	2	2	1	1	1	1	1
	6	4	2	1	1	1	1	1	1

Table 1: Minimum sample size (n) for MChSP-1 plan when the life time of the item follows Generalised exponential distribution

ρ	i	t/σ <sub>0</sub>							
β	1	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	1	4	2	2	2	1	1	1	1
	2	2	1	1	1	1	1	1	1
0.25	3	1	1	1	1	1	1	1	1
0.25	4	1	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	6	4	3	3	2	2	1	1
	2	3	2	1	1	1	1	1	1
0.10	3	2	1	1	1	1	1	1	1
0.10	4	1	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	7	5	4	3	2	2	2	2
	2	3	2	2	1	1	1	1	1
0.05	3	2	2	1	1	1	1	1	1
0.05	4	2	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	11	7	6	5	4	3	3	2
	2	5	3	3	2	2	1	1	1
0.01	3	3	3	2	1	1	1	1	1
	4	2	2	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1

Table 2: Minimum sample size (n) for MChSP-1 plan when the life time of the item follows Marshall – Olkin extended lomax distribution

ß	i	t/σ <sub>0</sub>							
β	1	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	1	6	4	3	2	1	1	1	1
	2	3	2	1	1	1	1	1	1
0.25	3	2	1	1	1	1	1	1	1
0.25	4	2	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	10	6	4	3	2	1	1	1
	2	5	3	2	1	1	1	1	1
0.10	3	3	2	1	1	1	1	1	1
	4	2	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
	1	12	8	5	4	3	2	1	1
	2	6	4	2	2	1	1	1	1
0.05	3	4	2	2	1	1	1	1	1
0.05	4	3	2	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
	1	18	11	8	6	4	3	2	2
	2	9	5	4	3	2	1	1	1
0.01	3	6	3	2	2	1	1	1	1
0.01	4	4	2	2	1	1	1	1	1
	5	3	2	1	1	1	1	1	1
	6	3	2	1	1	1	1	1	3

Table 3: Minimum sample size (n) for MChSP-1 plan when the life time of the item follows Marshall – Olkin extended exponential distribution

ß	i	t/σ <sub>0</sub>							
β	1	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	1	5	2	1	1	1	1	1	1
	2	3	1	1	1	1	1	1	1
0.25	3	2	1	1	1	1	1	1	1
0.25	4	1	1	1	1	1	1	1	1
	5	1	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	9	4	2	1	1	1	1	1
	2	4	2	1	1	1	1	1	1
0.10	3	3	1	1	1	1	1	1	1
0.10	4	2	1	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	1	1	1	1	1	1	1	1
	1	11	5	3	2	1	1	1	1
	2	5	2	1	1	1	1	1	1
0.05	3	3	1	1	1	1	1	1	1
0.05	4	3	1	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
	1	16	7	4	3	1	1	1	1
	2	8	3	2	1	1	1	1	1
0.01	3	5	2	1	1	1	1	1	1
	4	4	2	1	1	1	1	1	1
	5	3	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1

 Table 4: Minimum sample size (n) for MChSP-1 plan when the life time of the item follows Weibull distribution

β	i	t/σ <sub>0</sub>							
þ	1	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	1	11	5	3	2	1	1	1	1
	2	6	2	1	1	1	1	1	1
0.25	3	4	2	1	1	1	1	1	1
0.20	4	3	1	1	1	1	1	1	1
	5	2	1	1	1	1	1	1	1
	6	2	1	1	1	1	1	1	1
	1	18	8	4	3	1	1	1	1
	2	9	4	2	1	1	1	1	1
0.10	3	6	2	1	1	1	1	1	1
0.10	4	4	2	1	1	1	1	1	1
	5	3	2	1	1	1	1	1	1
	6	3	2	1	1	1	1	1	1
	1	23	10	6	4	2	1	1	1
	2	11	5	3	2	1	1	1	1
0.05	3	7	3	1	1	1	1	1	1
0.05	4	5	2	1	1	1	1	1	1
	5	4	2	1	1	1	1	1	1
	6	4	2	1	1	1	1	1	1
	1	33	15	8	5	2	1	1	1
	2	16	7	4	2	1	1	1	1
0.01	3	11	4	2	2	1	1	1	1
0.01	4	8	3	2	1	1	1	1	1
	5	6	3	2	1	1	1	1	1
	6	5	2	1	1	1	1	1	1

Table 5: Minimum sample size (n) for MChsp plan when the life time of the item follows Generalised Rayleigh distribution

β	i	t/σ <sub>0</sub>							
þ	1	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	1	27	5	3	2	1	1	1	1
	2	14	3	1	1	1	1	1	1
0.25	3	9	2	1	1	1	1	1	1
0.20	4	7	1	1	1	1	1	1	1
	5	6	1	1	1	1	1	1	1
	6	5	1	1	1	1	1	1	1
	1	43	9	5	3	2	1	1	1
	2	22	4	2	1	1	1	1	1
0.10	3	14	3	1	1	1	1	1	1
0.10	4	11	2	1	1	1	1	1	1
	5	9	2	1	1	1	1	1	1
	6	7	1	1	1	1	1	1	1
	1	54	11	6	4	2	2	2	1
	2	27	5	3	2	1	1	1	1
0.05	3	18	4	2	1	1	1	1	1
0.05	4	13	3	1	1	1	1	1	1
	5	11	2	1	1	1	1	1	1
	6	9	2	1	1	1	1	1	1
	1	78	16	8	6	4	3	2	2
	2	39	8	4	3	2	1	1	1
0.01	3	26	5	2	2	1	1	1	1
	4	19	4	2	1	1	1	1	1
	5	15	3	2	1	1	1	1	1
	6	13	2	1	1	1	1	1	1

Table 6: Minimum sample size (n) for MChSP-1plan when the life time of the item follows InverseRayleigh distribution

β	n	t/σ <sub>0</sub>	σ/σ <sub>0</sub>					
Р		0	2	4	6	8	10	12
	4	0.628	0.699420	0.913488	0.959977	0.977018	0.985101	0.989562
	2	0.942	0.665307	0.903871	0.955733	0.974597	0.983520	0.988441
	1	1.257	0.687176	0.913284	0.960671	0.977564	0.985477	0.989821
0.25	1	1.571	0.555381	0.866738	0.939217	0.965337	0.977571	0.984279
0.25	1	2.356	0.276888	0.719910	0.866792	0.923564	0.950537	0.965352
	1	3.141	0.116673	0.555588	0.772519	0.866819	0.913365	0.939255
	1	3.927	0.044072	0.402408	0.665382	0.797609	0.866770	0.906192
	1	4.712	0.015546	0.276888	0.555519	0.719910	0.812290	0.866792
	7	0.628	0.533444	0.853460	0.931003	0.960126	0.974072	0.981804
	4	0.942	0.448350	0.817438	0.913488	0.949854	0.967316	0.977018
	2	1.257	0.484305	0.835143	0.923041	0.955665	0.971175	0.979749
0.10	2	1.571	0.326390	0.753924	0.882584	0.931986	0.955679	0.968818
0.10	1	2.356	0.276888	0.719910	0.866792	0.923564	0.950537	0.965352
	1	3.141	0.116673	0.555588	0.772519	0.866819	0.913365	0.939255
	1	3.927	0.044072	0.402408	0.665382	0.797609	0.866770	0.906192
	1	4.712	0.015546	0.276888	0.555519	0.719910	0.812290	0.866792
	9	0.628	0.442656	0.815435	0.912146	0.949022	0.966786	0.976665
	4	0.942	0.448350	0.817438	0.913488	0.949854	0.967316	0.977018
	3	1.257	0.343904	0.764083	0.886951	0.934273	0.957085	0.969782
0.05	2	1.571	0.326390	0.753924	0.882584	0.931986	0.955679	0.968818
0.05	1	2.356	0.276888	0.719910	0.866792	0.923564	0.950537	0.965352
	1	3.141	0.116673	0.555588	0.772519	0.866819	0.913365	0.939255
	1	3.927	0.044072	0.402408	0.665382	0.797609	0.866770	0.906192
	1	4.712	0.015546	0.276888	0.555519	0.719910	0.812290	0.866792
	13	0.628	0.299448	0.743638	0.875458	0.927171	0.952371	0.966467
	7	0.942	0.242766	0.702188	0.853460	0.913880	0.943499	0.960126
	4	1.257	0.243609	0.699023	0.852267	0.913358	0.943200	0.959916
0.01	3	1.571	0.194607	0.656682	0.829550	0.899834	0.934293	0.953606
0.01	2	2.356	0.093542	0.528584	0.866792	0.853758	0.903794	0.932013
	1	3.141	0.116673	0.555588	0.772519	0.866819	0.913365	0.939255
	1	3.927	0.044072	0.402408	0.665382	0.797609	0.866770	0.906192
	1	4.712	0.015546	0.276888	0.555519	0.719910	0.812290	0.866792

Table 7 : Probability of acceptance for MChSP-1 plan with i = 2, when the life time of the item follows Generalised exponential distribution

β	n	t/σ <sub>0</sub>	$\sigma/\sigma_0$	_		-		
P		Ů	2	4	6	8	10	12
	2	0.628	0.382108	0.656145	0.769591	0.828505	0.864005	0.887565
	1	0.942	0.438454	0.702826	0.806591	0.858780	0.889525	0.909585
	1	1.257	0.312840	0.604710	0.736782	0.806424	0.848275	0.875833
0.25	1	1.571	0.222709	0.516211	0.669128	0.754123	0.806458	0.841357
0.23	1	2.356	0.097257	0.340700	0.516299	0.628525	0.702697	0.754178
	1	3.141	0.044957	0.222829	0.392160	0.516343	0.604889	0.669234
	1	3.927	0.022143	0.146333	0.295725	0.420383	0.516264	0.589359
	1	4.712	0.011600	0.097257	0.222789	0.340700	0.438269	0.516299
	3	0.628	0.244152	0.535336	0.676937	0.755083	0.803674	0.836539
	2	0.942	0.212341	0.505216	0.656145	0.740532	0.793064	0.828505
	1	1.257	0.312840	0.604710	0.736782	0.806424	0.848275	0.875833
0.10	1	1.571	0.222709	0.516211	0.669128	0.754123	0.806458	0.841357
0.10	1	2.356	0.097257	0.340700	0.516299	0.628525	0.702697	0.754178
	1	3.141	0.044957	0.222829	0.392160	0.516343	0.604889	0.669234
	1	3.927	0.022143	0.146333	0.295725	0.420383	0.516264	0.589359
	1	4.712	0.011600	0.097257	0.222789	0.340700	0.438269	0.516299
	3	0.628	0.244151	0.535336	0.676937	0.755083	0.803673	0.836539
	2	0.942	0.212341	0.505216	0.656145	0.740532	0.793064	0.828505
	2	1.257	0.116108	0.381761	0.552225	0.655885	0.723068	0.769405
0.05	1	1.571	0.222709	0.516211	0.669128	0.754123	0.806458	0.841357
0.05	1	2.356	0.097257	0.340700	0.516299	0.628525	0.702697	0.754178
	1	3.141	0.044957	0.222829	0.392160	0.516343	0.604889	0.669234
	1	3.927	0.022143	0.146333	0.295725	0.420383	0.516264	0.589359
	1	4.712	0.011600	0.097257	0.222789	0.340700	0.438269	0.516299
	5	0.628	0.097715	0.354833	0.522944	0.626731	0.695083	0.742945
	3	0.942	0.105149	0.365656	0.535336	0.639545	0.707698	0.755083
	3	1.257	0.044528	0.243831	0.416146	0.535024	0.617444	0.676695
0.01	2	1.571	0.063767	0.285518	0.460773	0.577112	0.655937	0.711701
0.01	2	2.356	0.015129	0.135121	0.285607	0.410075	0.505041	0.577192
	1	3.141	0.044957	0.222829	0.392160	0.516343	0.604889	0.669234
	1	3.927	0.022143	0.146333	0.295725	0.420383	0.516264	0.589359
	1	4.712	0.011600	0.097257	0.222789	0.340700	0.438269	0.516299

# Table 8 : Probability of acceptance for MChSP-1 plan with i = 2, when the life time of the item follows Marshall – Olkin extended lomax distribution

β	n	$t/\sigma_0$	$\sigma/\sigma_0$	1	1			
Р		<b>UU</b> <sub>0</sub>	2	4	6	8	10	12
	3	0.628	0.503647	0.745427	0.832148	0.875421	0.901139	0.918125
	2	0.942	0.454772	0.724681	0.821351	0.868873	0.896760	0.914995
	1	1.257	0.541647	0.789038	0.868400	0.905492	0.926619	0.940160
0.25	1	1.571	0.429234	0.726950	0.829445	0.877894	0.905510	0.923177
0.23	1	2.356	0.214830	0.571744	0.727016	0.804396	0.849177	0.877923
	1	3.141	0.094331	0.429403	0.622803	0.727050	0.789156	0.829509
	1	3.927	0.037412	0.309311	0.522183	0.648630	0.726990	0.778858
	1	4.712	0.013783	0.214830	0.429346	0.571744	0.664289	0.727016
	5	0.628	0.320865	0.613495	0.736474	0.801245	0.840796	0.867346
	3	0.942	0.313945	0.619473	0.745427	0.810423	0.849499	0.875421
	2	1.257	0.311750	0.629072	0.756739	0.821198	0.859338	0.884339
0.10	1	1.571	0.429234	0.726950	0.829445	0.877894	0.905510	0.923177
0.10	1	2.356	0.214830	0.571744	0.727016	0.804396	0.849177	0.877923
	1	3.141	0.094331	0.429403	0.622803	0.727050	0.789156	0.829509
	1	3.927	0.037412	0.309311	0.522183	0.648630	0.726990	0.778858
	1	4.712	0.013783	0.214830	0.429346	0.571744	0.664289	0.727016
	6	0.628	0.254442	0.555818	0.692515	0.766378	0.812059	0.842958
	4	0.942	0.216094	0.529357	0.676463	0.755883	0.804718	0.837553
	2	1.257	0.311750	0.629072	0.756739	0.821198	0.859338	0.884339
0.05	2	1.571	0.204358	0.538380	0.692373	0.772938	0.821229	0.853053
0.03	1	2.356	0.214830	0.571744	0.727016	0.804396	0.849177	0.877923
	1	3.141	0.094331	0.429403	0.622803	0.727050	0.789156	0.829509
	1	3.927	0.037412031	0.309311	0.522183	0.648630	0.726990	0.778858
	1	4.712	0.013783	0.214830	0.429346	0.571744	0.664289	0.727016
	9	0.628	0.122773	0.409812	0.573836	0.669522	0.730935	0.773393
	5	0.942	0.147519	0.451563	0.613495	0.704806	0.762176	0.801245
	4	1.257	0.105794	0.402322	0.576027	0.676215	0.739565	0.782761
	3	1.571	0.099552	0.401049	0.579271	0.681292	0.745263	0.788563
0.01	2	2.356	0.059866	0.344272	0.538473	0.652723	0.724558	0.772989
	1	3.141	0.094331	0.429403	0.622803	0.727050	0.789156	0.829509
	1	3.927	0.037412	0.309311	0.522183	0.648630	0.726990	0.778858
	1	4.712	0.013783	0.214830	0.429346	0.571744	0.664289	0.727016

Table 9 : Probability of acceptance for MChSP-1 plan with i = 2, when the life time of the item followsMarshall – Olkin extended exponential distribution

Г

ß		t/σ <sub>0</sub>	$\sigma/\sigma_0$					
β	n	<b>UO</b> <sub>0</sub>	2	4	6	8	10	12
	3	0.628	0.694253	0.923977	0.966655	0.981353	0.988101	0.991751
	1	0.942	0.718535	0.938086	0.973945	0.985675	0.990935	0.993745
	1	1.257	0.505275	0.883440	0.951916	0.973902	0.983606	0.988737
0.25	1	1.571	0.301724	0.809525	0.921986	0.958144	0.973910	0.982165
0.25	1	2.356	0.038911	0.560250	0.809613	0.898991	0.938029	0.958163
	1	3.141	0.001731	0.302013	0.650178	0.809658	0.883563	0.922040
	1	3.927	2.80479E	0.124146	0.469385	0.693209	0.809578	0.872525
	1	4.712	1.75E-07	0.038911	0.301917	0.560250	0.718284	0.809613
	4	0.628	0.615582	0.899973	0.955792	0.975216	0.984167	0.989016
	2	0.942	0.526682	0.880481	0.948621	0.971565	0.981955	0.987529
	1	1.257	0.505275	0.883440	0.951916	0.973902	0.983606	0.988737
0 10	1	1.571	0.301724	0.809525	0.921986	0.958144	0.973910	0.982165
0.10	1	2.356	0.038911	0.560250	0.809613	0.898991	0.938029	0.958163
	1	3.141	0.001731	0.302013	0.650178	0.809658	0.883563	0.922040
	1	3.927	2.80479E	0.124146	0.469385	0.693209	0.809578	0.872525
	1	4.712	1.75E-07	0.038911	0.301917	0.560250	0.718284	0.809613
	5	0.628	0.545280	0.876571	0.945049	0.969116	0.980248	0.986289
	2	0.942	0.526682	0.880481	0.948621	0.971565	0.981955	0.986289
	1	1.257	0.505275	0.883440	0.951916	0.973902	0.983606	0.988737
	1	1.571	0.301724	0.809525	0.921986	0.958144	0.973910	0.982165
0.05	1	2.356	0.038911	0.560250	0.809613	0.898991	0.938029	0.958163
	1	3.141	0.001731	0.302013	0.650178	0.809658	0.883563	0.922040
	1	3.927	2.80479E- 05	0.124146	0.469385	0.693209	0.809578	0.872525
	1	4.712	1.75E-07	0.038911	0.301917	0.560250	0.718284	0.809613
	8	0.628	0.374894	0.809663	0.913505	0.951039	0.968583	0.978153
	3	0.942	0.388440	0.826606	0.923977	0.957661	0.973057	0.981353
	2	1.257	0.274590	0.782497	0.906399	0.948536	0.967494	0.977605
0.01	1	1.571	0.301724	0.809525	0.921986	0.958144	0.973910	0.982165
0.01	1	2.356	0.038911	0.560250	0.809613	0.898991	0.938029	0.958163
	1	3.141	0.001731	0.302013	0.650178	0.809658	0.883563	0.922040
	1	3.927	2.80479E	0.124146	0.469385	0.693209	0.809578	0.872525
	1	4.712	1.75E-07	0.038911	0.301917	0.560250	0.718284	0.809613

Table 10: Probability of acceptance for MChSP-1 plan with i = 2, when the life time of item follows Weibull distribution

Ø	_	<i>t/-</i>	σ/σ <sub>0</sub>					
β	n	t/σ <sub>0</sub>	2	4	6	8	10	12
	6	0.628	0.715843	0.926269	0.967154	0.981517	0.988169	0.991784
	2	0.942	0.755188	0.941957	0.974788	0.985951	0.991050	0.993800
	1	1.257	0.751132	0.945408	0.976931	0.987289	0.991948	0.994440
0.25	1	1.571	0.606742	0.911228	0.963085	0.979844	0.987294	0.991252
0.25	1	2.356	0.249566	0.783576	0.911269	0.952534	0.970482	0.979853
	1	3.141	0.059782	0.606985	0.832716	0.911290	0.945464	0.963110
	1	3.927	0.008342	0.415734	0.728753	0.854825	0.911253	0.940401
	1	4.712	0.000696	0.249566	0.606904	0.783576	0.867360	0.911269
	9	0.628	0.603555	0.891391	0.951130	0.972402	0.982306	0.987701
	4	0.942	0.573721	0.887411	0.950225	0.972101	0.982180	0.987640
	2	1.257	0.572777	0.894145	0.954431	0.974747	0.983962	0.988911
0.10	1	1.571	0.606742	0.911228	0.963085	0.979844	0.987294	0.991252
0.10	1	2.356	0.249566	0.783576	0.911269	0.952534	0.970482	0.979853
	1	3.141	0.059782	0.606985	0.832716	0.911290	0.945464	0.963110
	1	3.927	0.008342	0.415734	0.728753	0.854825	0.911253	0.940401
	1	4.712	0.000696	0.249566	0.606904	0.783576	0.867360	0.911269
	11	0.628	0.537088	0.868799	0.940588	0.966371	0.978416	0.984989
	5	0.942	0.499340	0.861303	0.938174	0.965249	0.977775	0.984574
	3	1.257	0.438899	0.845807	0.932466	0.962368	0.976041	0.983413
0.05	2	1.571	0.384136	0.831449	0.927661	0.960119	0.974756	0.982582
0.05	1	2.356	0.249566	0.783576	0.911269	0.952534	0.970482	0.979853
	1	3.141	0.059782	0.606985	0.832716	0.911290	0.945464	0.963110
	1	3.927	0.008342	0.415734	0.728753	0.854825	0.911253	0.940401
	1	4.712	0.000696	0.249566	0.606904	0.783576	0.867360	0.911269
	16	0.628	0.396374	0.814439	0.914698	0.951449	0.968757	0.978239
	7	0.942	0.375777	0.811190	0.914506	0.951685	0.969023	0.978469
	4	1.257	0.335859	0.800070	0.911006	0.950146	0.968184	0.977946
0.01	2	1.571	0.384136	0.831449	0.927661	0.960119	0.974756	0.982582
0.01	1	2.356	0.249566	0.783576	0.911269	0.952534	0.970482	0.979853
	1	3.141	0.059782	0.606985	0.832716	0.911290	0.945464	0.963110
	1	3.927	0.008342	0.415734	0.728753	0.854825	0.911253	0.940401
	1	4.712	0.000696	0.249566	0.606904	0.783576	0.867360	0.911269

Table 11:Probability of acceptance for MChSP-1 plan with i = 2, when the life time of item follows Rayleigh distribution

β	n	t/σ <sub>0</sub>	σ/σ₀								
			2	4	6	8	10	12			
0.25	3	0.628	0.999448	1.000000	1.000000	1.000000	1.000000	1.000000			
	1	0.942	0.966256	1.000000	1.000000	1.000000	1.000000	1.000000			
	1	1.257	0.903924	0.999960	1.000000	1.000000	1.000000	1.000000			
	1	1.571	0.720531	0.998464	1.000000	1.000000	1.000000	1.000000			
	1	2.356	0.267213	0.935457	0.998468	0.999990	1.000000	1.000000			
	1	3.141	0.086407	0.720870	0.972038	0.998470	0.999960	1.000000			
	1	3.927	0.030328	0.459924	0.879354	0.983511	0.998466	0.999912			
	1	4.712	0.011965	0.267213	0.720757	0.935457	0.988574	0.998468			
0.10	4	0.628	0.999134	1.000000	1.000000	1.000000	1.000000	1.000000			
	2	0.942	0.955266	1.000000	1.000000	1.000000	1.000000	1.000000			
	1	1.257	0.818409	0.999920	1.000000	1.000000	1.000000	1.000000			
	1	1.571	0.720531	0.998464	1.000000	1.000000	1.000000	1.000000			
	1	2.356	0.267213	0.935457	0.998468	0.999990	1.000000	1.000000			
	1	3.141	0.086407	0.720870	0.972038	0.998470	0.999960	1.000000			
	1	3.927	0.030328	0.459924	0.879354	0.983511	0.998466	0.999912			
	1	4.712	0.011965	0.267213	0.720757	0.935457	0.988574	0.998468			
0.05	5	0.628	0.998937	1.000000	1.000000	1.000000	1.000000	1.000000			
	2	0.942	0.944398	1.000000	1.000000	1.000000	1.000000	1.000000			
	1	1.257	0.741469	0.999880	1.000000	1.000000	1.000000	1.000000			
	1	1.571	0.529444	0.996930	0.999999	1.000000	1.000000	1.000000			
	1	2.356	0.267213	0.935457	0.998468	0.999990	1.000000	1.000000			
	1	3.141	0.086407	0.720870	0.972038	0.998470	0.999960	1.000000			
	1	3.927	0.030328	0.459924	0.879354	0.983511	0.998466	0.999912			
	1	4.712	0.011965	0.267213	0.720757	0.935457	0.988574	0.998468			
0.01	8	0.628	0.998465	1.000000	1.000000	1.000000	1.000000	1.000000			
	3	0.942	0.912498	1.000000	1.000000	1.000000	1.000000	1.000000			
	2	1.257	0.671702	0.999840	1.000000	1.000000	1.000000	1.000000			
	1	1.571	0.391405	0.995398	0.999999	1.000000	1.000000	1.000000			
	1	2.356	0.087850	0.875607	0.996938	0.999980	1.000000	1.000000			
	1	3.141	0.086407	0.720870	0.972038	0.998470	0.999960	1.000000			
	1	3.927	0.0303284	0.459924	0.879354	0.983511	0.998466	0.999912			
	1	4.712	0.011965	0.267213	0.720757	0.935457	0.988574	0.998468			

Table 12 : Probability of acceptance for MChSP-1 plan with i = 2, when the life time of the item follows Inverse Rayleigh distribution

# VII. EXAMPLES

Suppose that the experimenter is interested in establishing that the true unknown average life is at least 1000 hours. It is desired to stop the experiment at 628 hours with  $\beta$ = 0.25. Based on consumer's risk values and the time termination ratio, the minimum sample size is determined using the Modified chain sampling plan for truncated life test. Following are the results obtained when the lifetime of the test items follows the Rayleigh, Generalized exponential distribution, Weibull distribution, Inverse-Rayleigh Distribution, Marshall – Olkin extended exponential distribution and Marshall – Olkin extended lomax distribution.

7.1 Generalized exponential distribution: Let the distribution followed be Generalized Exponential, when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 1 is 4. If during 628

hours no failures out of 4 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours, and if more than one failure occurs in the i preceeding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 7, one can observe that the probability of acceptance for this sampling is 0.699420 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 0.989562 when the ratio of the unknown average life is 12.

7.2 Marshall – Olkin extended lomax distribution : Let the distribution followed be Marshall – Olkin Extended Lomax, when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 2 is 2. If during 628 hours no failures out of 2 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours ,and if more than one failure occurs in the i preceding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 8, one can observe that the probability of acceptance for this sampling is 0.382108 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 0.887565 when the ratio of the unknown average life is 12.

7.3 Marshall – Olkin extended exponential distribution : Let the distribution followed be Marshall – Olkin Extended Exponential , when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 3 is 3. If during 628 hours no failures out of 3 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours ,and if more than one failure occurs in the i preceeding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 9, one can observe that the probability of acceptance for this sampling is 0.503647 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 0.918125 when the ratio of the unknown average life is 12.

7.4 Weibull distribution : Let the distribution followed be Weibull, when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 4 is 3. If during 628 hours no failures out of 3 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours, and if more than one failure occurs in the i preceeding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 10, one can observe that the probability of acceptance for this sampling is 0.694253 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 0.991751 when the ratio of the unknown average life is 12.

7.5 Rayleigh distribution : Let the distribution followed be Rayleigh, Let the distribution followed be Weibull, when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 5 is 6. If during 628 hours no failures out of 6 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours ,and if more than one failure occurs in the i preceding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 11, one can observe that the probability of acceptance for this sampling is 0.718535 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 0.993745 when the ratio of the unknown average life is 12.

7.6 Inverse-Rayleigh distribution: Let the distribution followed be Inverse-Rayleigh, Let the distribution followed be Weibull, when the acceptance criteria (previous lots number) is predefined as i = 2, the required *n* from Table 6 is 3. If during 628 hours no failures out of 3 are observed then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours ,and if more than one failure occurs in the i preceeding samples, reject the lot and otherwise accept the lot and repeat the experiment. From the Table 12, one can observe that the probability of acceptance for this sampling is 0.999448 when  $\sigma/\sigma_0 = 2$ . For the same measurements and plan parameters, the probability of acceptance is 1.000000 when the ratio of the unknown average life is 12.

Lifetime distribution	n	σ/σ					
		2	4	6	8	10	12
Generalized Exponential distribution	4	0.699420	0.913488	0.959977	0.977018	0.985101	0.989562
Marshall – Lomax extended Exponential distribution	2	0.382108	0.656145	0.769591	0.828505	0.864005	0.887565
Marshall – Olkin Extended Exponential distribution	3	0.503647	0.745427	0.832148	0.875421	0.901139	0.918125
Weibull distribution	3	0.694253	0.923977	0.966655	0.981353	0.988101	0.991751
Rayleigh distribution	6	0.715843	0.926269	0.967154	0.981517	0.988169	0.991784
Inverse Rayleigh distribution	3	0.999448	1.000000	1.000000	1.000000	1.000000	1.000000

The above example for different lifetime distributions is tabulated below:

## **VIII. CONCLUSIONS**

In this paper, designing a Modified Chain sampling plan (MChSP-1) for the truncated life test is presented. The minimum sample size and the probability of acceptance are calculated, for various values of test termination ratios, assuming that the lifetime of an item follows different distributions. When all the above tables (Table 7 to Table 12) are compared, it is observed that the operating characteristic values of Inverse Rayleigh distribution increases disproportionately and reaches the maximum value 1 when  $\sigma/\sigma_0$  is greater than 2 with n = 3. Generally speaking, it applies to all life models so long as the life distribution can be obtained and is a versatile sampling plan that can be conveniently used in costly and destructive testing to save the time and cost of life test experiments.

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